Buffered Probability of Exceedance: Mathematical Properties and Applications

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The probability of exceedance (POE) is frequently used to measure uncertainties in outcomes. For instance, POE is used to estimate probability that assets of a company fall below liabilities. POE measures only the frequency of outcomes and ignores magnitude of outcomes. POE counts outcomes exceeds the threshold, and it "does not worry" about the amount by which each outcome exceeds the threshold. POE is lumping together all threshold exceedance events, potentially "hiding" quite large and very troublesome outcomes. Moreover, POE has poor mathematical properties when used to characterize discrete distributions of random values, e.g., when distributions are defined by previously observed historical data. POE for discrete distributions is a discontinuous function of control variables, making it difficult to analyze and optimize.

This paper investigates a new probabilistic characteristic called b*uffered probability of exceedance* (bPOE). With bPOE, it is possible to count outcomes similar to a threshold value, rather than only outcomes exceeding the threshold. To be more precise, bPOE counts tail outcomes averaging to some specific threshold value. For instance, 4% of land-falling hurricanes in US have cumulative damage exceeding $50 billion (i.e., POE = 0.04 for threshold=$50 billion). It is estimated, that the average damage from the worst 10% of hurricanes is $50 billion. In terms of bPOE, we say bPOE=0.1 for threshold=$50 billion. bPOE shows that largest damages having magnitude around $50 billion have frequency 10%. bPOE can be considered as an important supplement to POE. We think that bPOE should be routinely calculated together with POE. This example shows that bPOE exceeds POE, which is why it is called Buffered Probability of Exceedance. The positive difference, bPOE-POE, can be interpreted as some "buffer." The bPOE concept was recently developed as an extension of Buffered Probability of Failure (introduced by Rockafellar and Royset). bPOE has been derived from Conditional Value-at-Risk (CVaR) characteristic of uncertainty. Actually, bPOE is an inverse function of CVaR and it inherits a majority of the exceptional mathematical properties of CVaR (which is a so called "coherent measure of risk"). Similar to CVaR, minimization of bPOE can be reduced to convex and Linear Programming.

We will discuss two applications of bPOE concept. The first application considers the Cash Matching of a Bond Portfolio. We minimize bPOE that assets exceed liabilities. The second application uses bPOE in data mining. Currently, the Area Under the Receiver Operating Characteristics Curve (AUC) is standardly used to evaluate classification models. AUC can be presented as the probability that some discrete random value is below zero. We explored so called Buffered AUC (bAUC) as a counterpart of the standard AUC.

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1. Norton, M. and S. Uryasev. Maximization of AUC and Buffered AUC in Classification. Research Report 2014-2 , ISE Dept., University of Florida, October 2014 (http://www.ise.ufl.edu/uryasev/files/2015/02/bAUC_workingPaper.pdf).

2. Uryasev S. Buffered Probability of Exceedance and Buffered Service Level: Definitions and Properties. Research Report 2014-3 , ISE Dept., University of Florida, October 2014 (http://www.ise.ufl.edu/uryasev/files/2011/08/bPOE_bSL_final.pdf)

3. Mafusalov, A. and S. Uryasev. Buffered Probability of Exceedance: Mathematical Properties and Optimization Algorithms. Research Report 2014-1 , ISE Dept., University of Florida, October 2014 (http://www.ise.ufl.edu/uryasev/files/2011/08/buffered_probability_of_exceedance.pdf).